

VIEWS AND INCREMENTAL VIEW MAINTENANCE

CSE 4/562: Database Systems | Lecture 17

```
SELECT l.partkey
FROM lineitem l, orders o
WHERE l.orderkey = o.orderkey AND o.orderdate > DATE(NOW() - '1 Month')
ORDER BY l.shipdate DESC LIMIT 10
```

—

```
SELECT l.supkey, COUNT(*)
FROM lineitem l, orders o
WHERE l.orderkey = o.orderkey AND o.orderdate > DATE(NOW() - '1 Month')
GROUP BY l.supkey
```

—

```
SELECT l.partkey, COUNT(*)
FROM lineitem l, orders o
WHERE l.orderkey = o.orderkey AND o.orderdate > DATE(NOW() - '1 Month')
GROUP BY l.partkey
```

```
CREATE VIEW salesSinceLastMonth AS
  SELECT l.*
  FROM lineitem l, orders o
  WHERE l.orderkey = o.orderkey
  AND o.orderdate > DATE(NOW() - '1 Month')
```

```
—
SELECT l.partkey FROM salesSinceLastMonth
ORDER BY l.shipdate DESC LIMIT 10
```

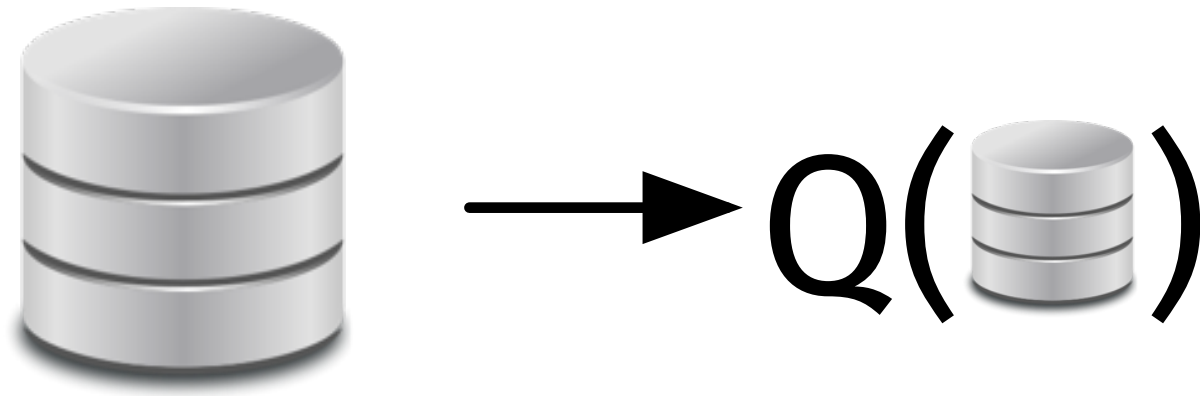
```
SELECT l.supkey, COUNT(*) FROM salesSinceLastMonth
GROUP BY l.supkey
```

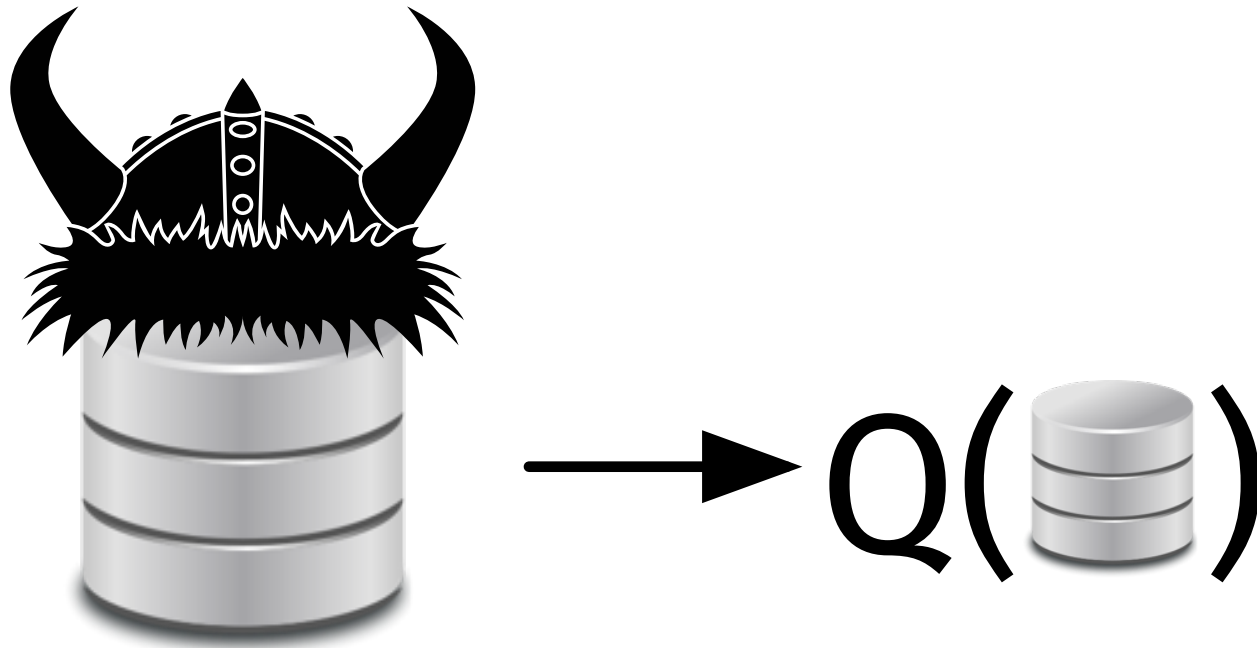
```
SELECT l.partkey, COUNT(*) FROM salesSinceLastMonth
GROUP BY l.partkey
```

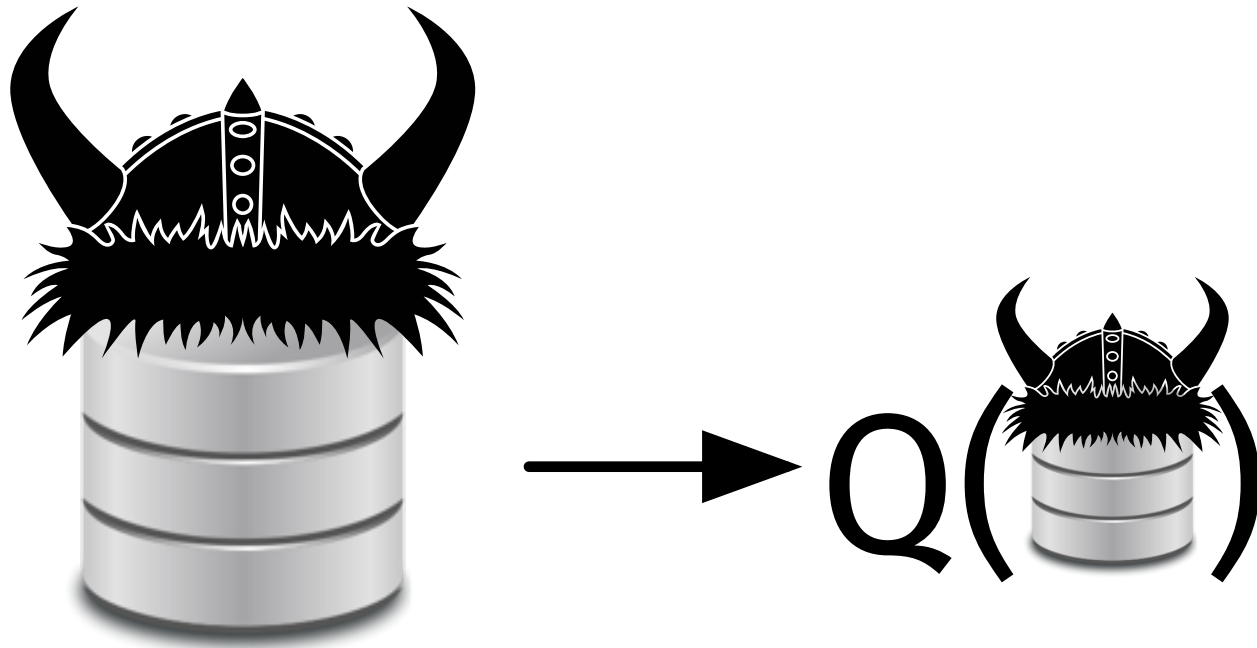
**Views exist to be queried
frequently**

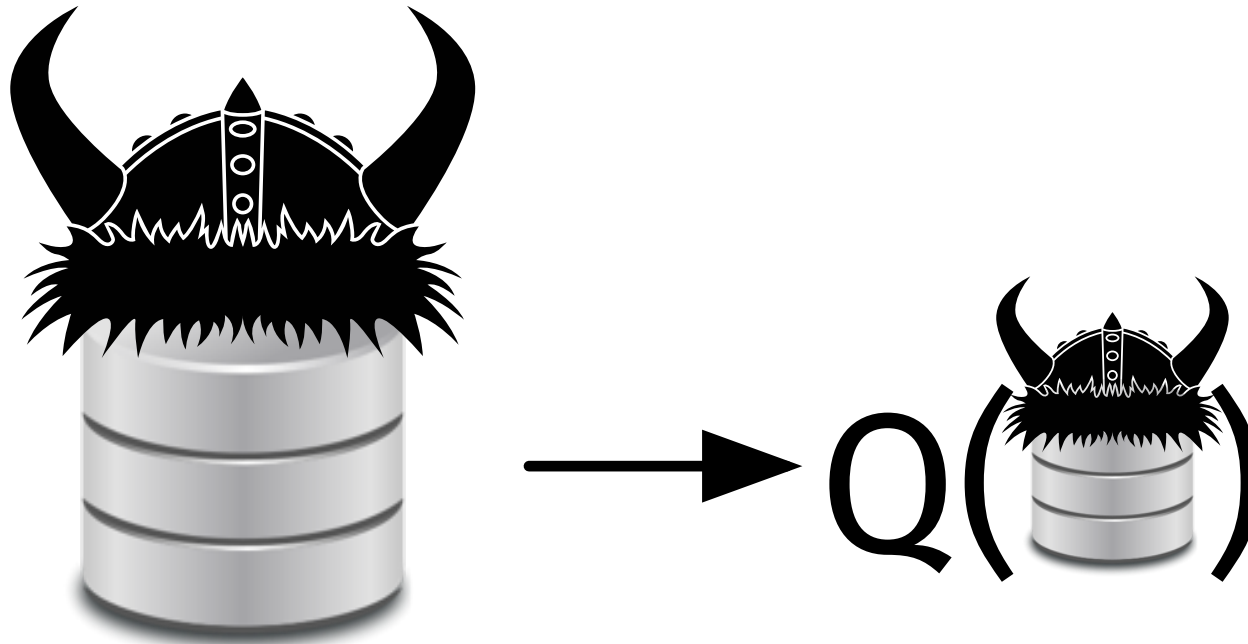
Idea: Pre-compute and save the view's contents
(like an index)











When the base data changes, the view needs to be updated too!

Initial Setup

Given: D_0

Initial Setup

Given: D_0

Compute: $VIEW_0 \leftarrow Q(D_0)$

Initial Setup

Given: D_0

Compute: $VIEW_0 \leftarrow Q(D_0)$

After an update

Given: D_1

Initial Setup

Given: D_0

Compute: $VIEW_0 \leftarrow Q(D_0)$

After an update

Given: D_1

Idea 1: Recompute the view.

Initial Setup

Given: D_0

Compute: $VIEW_0 \leftarrow Q(D_0)$

After an update

Given: D_1

Compute: $VIEW_1 \leftarrow Q(D_1)$

Idea 1: Recompute the view.

$Q(D_0)$ and $Q(D_1)$
are doing (mostly) the
same work

Initial Setup

Given: D_0

Initial Setup

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Compute: $VIEW_0 \leftarrow Q(D_0)$

Initial Setup

Given: D_0

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After an update

Given: δ_1 such that $D_1 = D_0 + \delta_1$

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Given: δ_1 such that $D_1 = D_0 + \delta_1$

Compute: $VIEW_1 \leftarrow Q(D_1)$

Initial Setup

Given: D_0

Compute: $VIEW_0 \leftarrow Q(D_0)$

After an update

Given: δ_1 such that $D_1 = D_0 + \delta_1$

Compute: $VIEW_1 \leftarrow \cancel{Q(D_1)} Q(D_0 + \delta_1)$

Initial Setup

Given: D_0

Compute: $VIEW_0 \leftarrow Q(D_0)$

After an update

Given: δ_1 such that $D_1 = D_0 + \delta_1$

Compute: $VIEW_1 \leftarrow \cancel{Q(D_1)} \quad \cancel{Q(D_0 + \delta_1)} \quad \Delta Q(D_0, \delta_1)$

Is there an operation “+” and a function $\Delta Q(D, \delta)$ such that...

1. $Q(D + \delta) = Q(D) + \Delta Q(D, \delta)$
2. $\Delta(D, \delta)$ is “cheap”
3. ‘+’ is “cheap”

Given

- $D = \{1, 2, 3, 4\}$
- $\delta = \{5\}$
- $Q(D) = \text{SUM}(D)$

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Naive Approach

$$\text{SUM}(D \cup \delta) = 1 + 2 + 3 + 4 + 5$$

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$$O(|D| + |\delta|)$$

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Naive Approach

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$$O(|D| + |\delta|)$$

Incremental Approach

$$\text{SUM}(D \cup \delta) = \text{SUM}(D) + 5 = 10 + 5$$

Given

- $D = \{1, 2, 3, 4\}$
- $\delta = \{5\}$
- $Q(D) = \text{SUM}(D)$

Naive Approach

$$\text{SUM}(D \cup \delta) = 1 + 2 + 3 + 4 + 5$$

$$O(|D| + |\delta|)$$

Incremental Approach

$$\text{SUM}(D \cup \delta) = \text{SUM}(D) + 5 = 10 + 5$$

$$O(|\delta|)$$

Given

- $R = \{A, B, C\}$ $S = \{X, Y\}$
- $\delta_R = \{D\}$
- $Q(D) = \text{COUNT}(R \times S)$

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$\text{COUNT}(D \cup \delta) = \text{COUNT}(AX, AY, BX, BY, CX, CY, \underline{DX}, \underline{DY})$

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- $R = \{A, B, C\}$ $S = \{X, Y\}$
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Naive Approach

$$\text{COUNT}(D \cup \delta) = \text{COUNT}(AX, AY, BX, BY, CX, CY, \underline{DX}, \underline{DY}) \quad O((|R| + |\delta_R|) \cdot |S|)$$

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Incremental Approach

$$\text{COUNT}(D \cup \delta) = \text{COUNT}(D) + \text{COUNT}(\delta) = 6 + \text{COUNT}(\underline{DX}, \underline{DY})$$

Given

- $R = \{A, B, C\}$ $S = \{X, Y\}$
- $\delta_R = \{D\}$
- $Q(D) = \text{COUNT}(R \times S)$

Naive Approach

$$\text{COUNT}(D \cup \delta) = \text{COUNT}(AX, AY, BX, BY, CX, CY, \underline{DX}, \underline{DY}) \quad O((|R| + |\delta_R|) \cdot |S|)$$

Incremental Approach

$$\text{COUNT}(D \cup \delta) = \text{COUNT}(D) + \text{COUNT}(\delta) = 6 + \text{COUNT}(\underline{DX}, \underline{DY}) \quad O(|\delta_R| \cdot |S|)$$

+ is like \cup

\times is like \bowtie

Are these types of pattern common?

Semiring: $\langle \mathbb{S}, +, \times, 0, 1 \rangle$

Any set of 'things' \mathbb{S} (where $a, b, c \in \mathbb{S}$) such that...

$$a + b = c$$

$$a \times b = c$$

$$a + 0 = a$$

$$a \times 1 = a$$

$$a \times 0 = 0$$

$$a \times (b + c) = (a \times b) + (a \times c)$$

Semiring: $\langle \mathbb{S}, +, \times, 0, 1 \rangle$

Any set of ‘things’ \mathbb{S} (where $a, b, c \in \mathbb{S}$) such that...

“Closed”

$$a + b = c$$

$$a \times b = c$$

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Additive, Multiplicative Identities

$$a + 0 = a$$

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Addition distributes over multiplication

Ring: $\langle \mathbb{S}, +, \times, 0, 1, - \rangle$

A semiring with an additive inverse

$$a + (-a) = 0$$



THE TANGENT ENDS NOW!

Initial Setup

Given: D_0

Compute: $VIEW_0 \leftarrow Q(D_0)$

After an update

Given: D_1

Compute: $VIEW_1 += \Delta Q(D_0, \delta)$

What are “+”, δ , and ΔQ ?

What is δ_R ?

What is δ_R ?

- Insertions
- Deletions
- Updates

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- Insertions
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δ_R is a set/bag of insertions and a set/bag of deletions

$$R_i = R_{i-1} - (\delta_{R-}) \cup (\delta_{R+})$$

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Caveats

- '-' needs to account for multiplicities
- Generally we compute insertions and deletions separately

Let's focus on insertions for now.

Given

$$\text{VIEW} = Q(R, S, \dots)$$

We want...

$$\Delta Q \text{ such that... } \text{VIEW}_i = \text{VIEW}_{i-1} \cup \Delta Q(R_{i-1}, \delta_{R,i}, S_{i-1}, \delta_{S,i}, \dots)$$

Given

$$\text{VIEW} = Q(R, S, \dots)$$

We want...

$$\Delta Q \text{ such that... } \text{VIEW}_i = \text{VIEW}_{i-1} \cup \Delta Q(R_{i-1}, \delta_{R,i}, S_{i-1}, \delta_{S,i}, \dots)$$

We want to compute δ_{VIEW} in terms of Q

Suppose that $Q = \sigma(R)$

Also suppose that we know δ_R

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... then we want

$$\sigma(R) \cup ??? = \sigma(R \cup \delta_R)$$

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$$\sigma(R) \cup \sigma(\delta_R) = \sigma(R \cup \delta_R)$$

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$$\sigma(R) \cup \sigma(\delta_R) = \sigma(R \cup \delta_R)$$

—

$$\Delta\sigma(R) = \sigma(\delta_R)$$

Suppose that $Q = \sigma(Q'(\dots))$

Also suppose that we know $\Delta Q'(\dots)$

—

... then we want

$$\sigma(R) \cup \sigma(\Delta Q'(\dots)) = \sigma(R \cup \Delta Q'(\dots))$$

—

$$\Delta\sigma(Q'(\dots)) = \sigma(\Delta Q'(\dots))$$

Suppose that $Q = \pi(R)$

Also suppose that we know δ_R

—

... then we want

$$\pi(R) \cup ??? = \pi(R \cup \delta_R)$$

Suppose that $Q = \pi(R)$

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$$\pi(R) \cup \pi(\delta_R) = \pi(R \cup \delta_R)$$

—

$$\Delta\pi(R) = \pi(\delta_R)$$

Suppose that $Q = R \cup S$

Also suppose that we know δ_R, δ_S

—

... then we want

$$(R \cup S) \cup ??? = (R \cup \delta_R) \cup (S \cup \delta_S)$$

Suppose that $Q = R \cup S$

Also suppose that we know δ_R, δ_S

—

... then we want

$$(R \cup S) \cup (\delta_R \cup \delta_S) = (R \cup \delta_R) \cup (S \cup \delta_S)$$

—

$$\Delta(R \cup S) = \delta_R \cup \delta_S$$

Suppose that $Q = R \bowtie S$

Also suppose that we know δ_R, δ_S

—

Suppose that $Q = R \bowtie S$

Also suppose that we know δ_R, δ_S

—

$(R \cup \delta_R) \bowtie (S \cup \delta_S)$

Suppose that $Q = R \bowtie S$

Also suppose that we know δ_R, δ_S

—

$$(R \cup \delta_R) \bowtie (S \cup \delta_S)$$

$$(R \bowtie (S \cup \delta_S)) \cup (\delta_R \bowtie (S \cup \delta_S))$$

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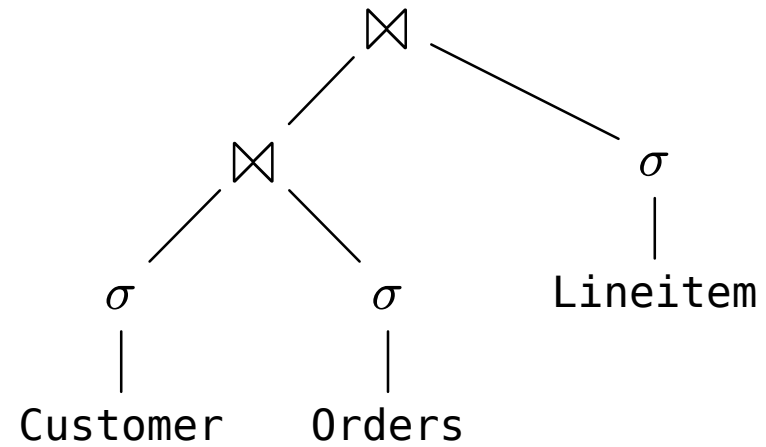
$$(R \bowtie S) \cup (R \bowtie \delta_S) \cup (\delta_R \bowtie S) \cup (\delta_R \bowtie \delta_S)$$

—

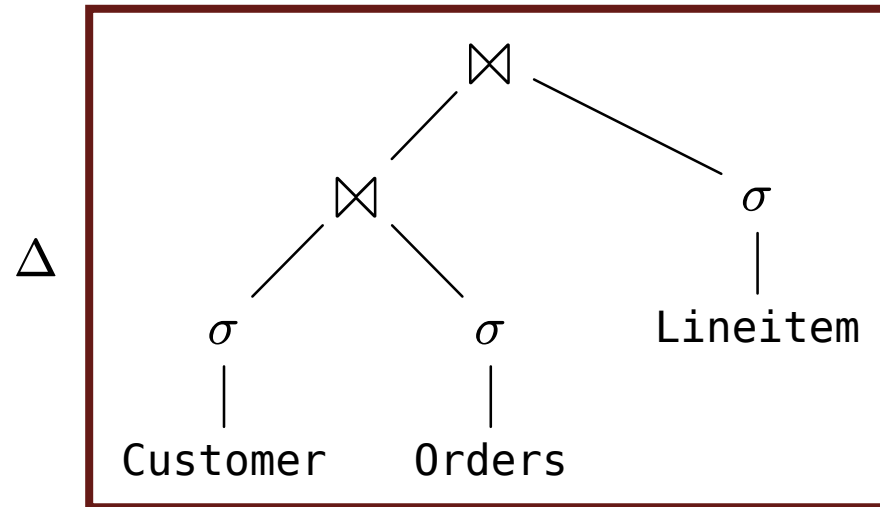
$$\Delta(R \bowtie S) = (R \bowtie \delta_S) \cup (\delta_R \bowtie S) \cup (\delta_R \bowtie \delta_S)$$

Example

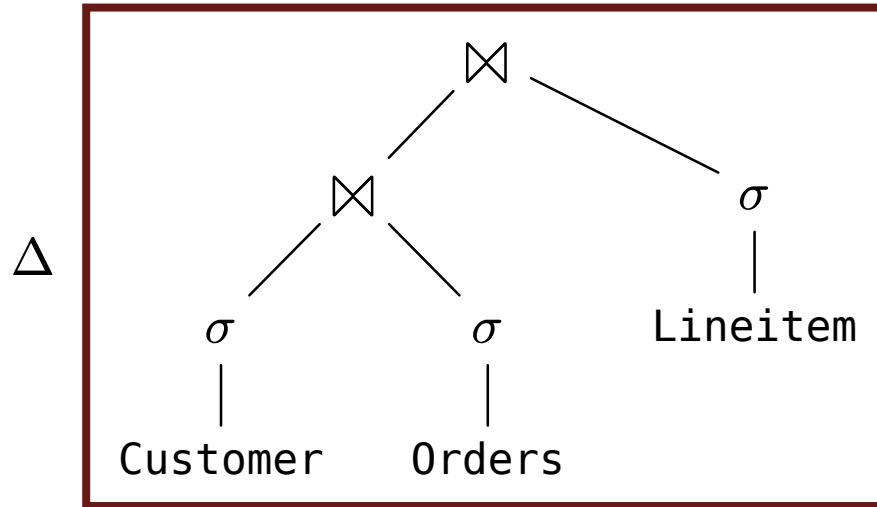
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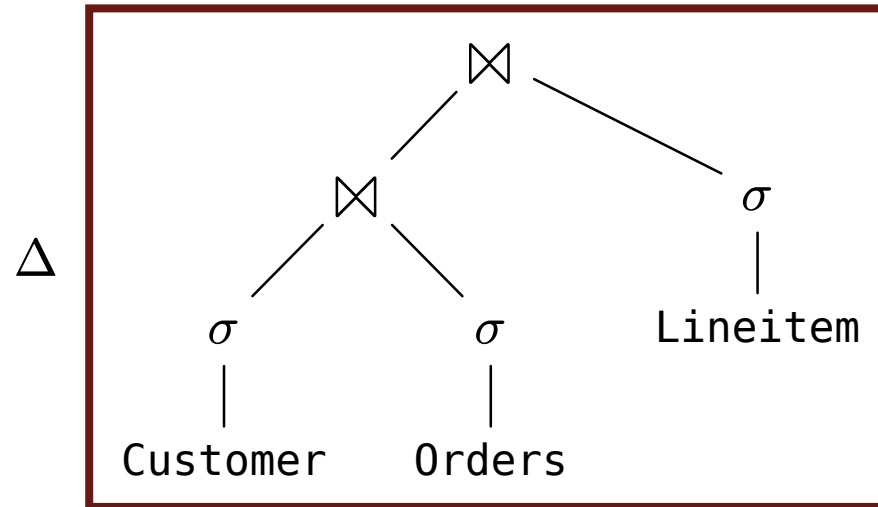


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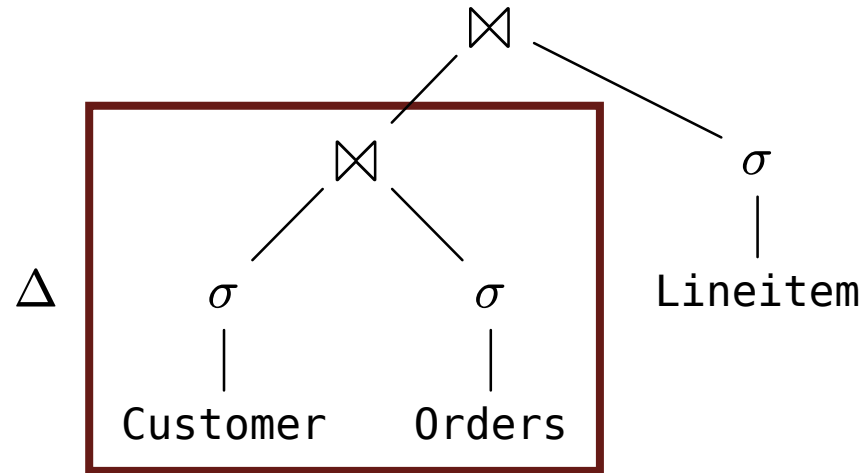
$$\Delta[(\sigma(C) \bowtie \sigma(O)) \bowtie \sigma(L)]$$

What is ΔQ with respect to a change in Lineitem?



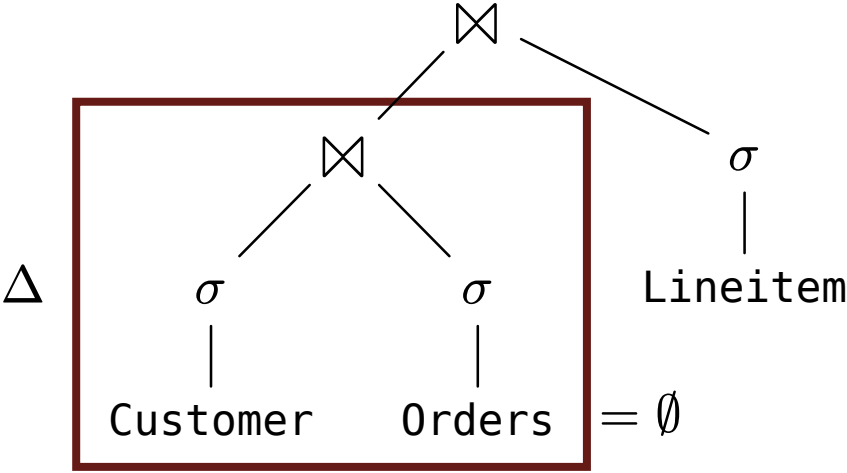
$$\Delta[(\sigma(C) \bowtie \sigma(O))] \bowtie \sigma(L) \cup (\sigma(C) \bowtie \sigma(O)) \bowtie \Delta[\sigma(L)] \cup \Delta[(\sigma(C) \bowtie \sigma(O))] \bowtie \Delta[\sigma(L)]$$

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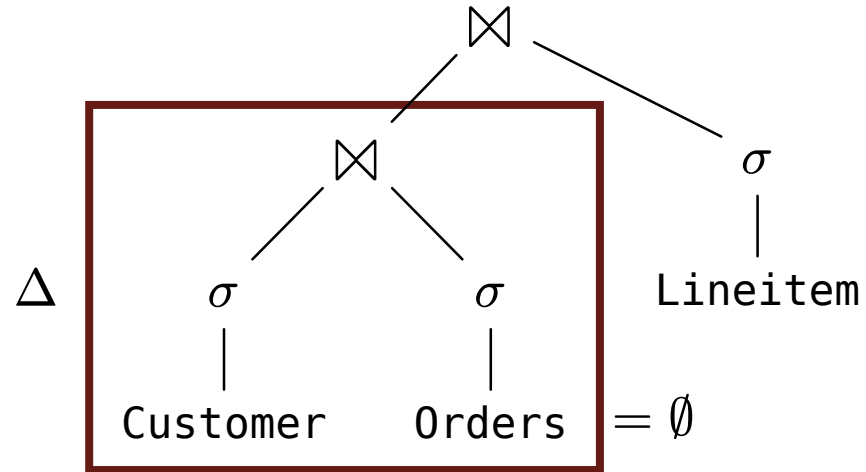
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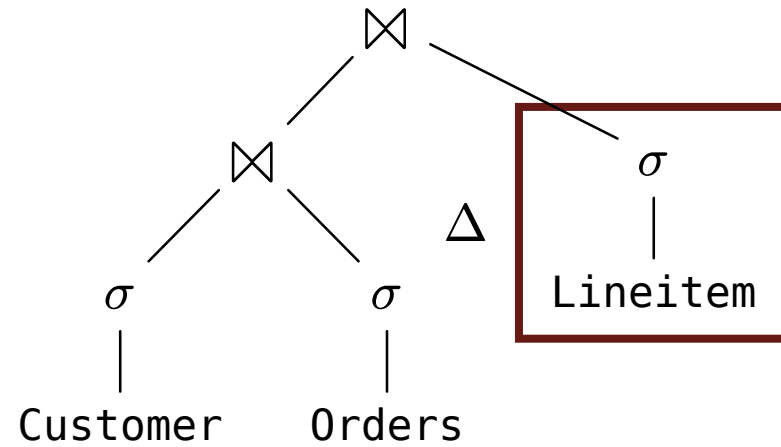
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What is ΔQ with respect to a change in Lineitem?



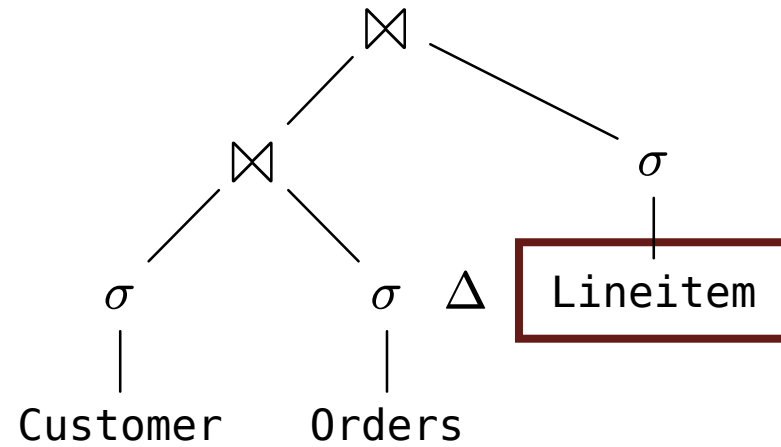
$$(\sigma(C) \bowtie \sigma(O)) \bowtie \Delta[\sigma(L)]$$

What is ΔQ with respect to a change in Lineitem?



$$(\sigma(C) \bowtie \sigma(O)) \bowtie \sigma(\Delta[L])$$

What is ΔQ with respect to a change in Lineitem?



$$(\sigma(C) \bowtie \sigma(O)) \bowtie \sigma(\delta_L)$$

Multisets

$\{1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 4, 4, 4, 4, 4, 4, 5\}$

$\{1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 4, 4, 4, 4, 4, 4, 5\}$

(not compact)

$\{1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 4, 4, 4, 4, 4, 4, 5\}$

(not compact)

$\{1 \rightarrow 3; 2 \rightarrow 5; 3 \rightarrow 2; 4 \rightarrow 6; 5 \rightarrow 1\}$

$\{1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 4, 4, 4, 4, 4, 4, 5\}$

(not compact)

$\{1 \rightarrow 3; 2 \rightarrow 5; 3 \rightarrow 2; 4 \rightarrow 6; 5 \rightarrow 1\}$

Multiset Representation: Tuple \rightarrow # of occurrences

$\{1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 4, 4, 4, 4, 4, 4, 5\}$

(not compact)

$\{1 \rightarrow 3; 2 \rightarrow 5; 3 \rightarrow 2; 4 \rightarrow 6; 5 \rightarrow 1\}$

Multiset Representation: Tuple \rightarrow ~~# of occurrences~~ multiplicity

How does union work with multisets?

$$\{\langle A \rangle \rightarrow 1, \langle B \rangle \rightarrow 3\} \cup \{\langle B \rangle \rightarrow 2, \langle C \rangle \rightarrow 4\} =$$

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How does union work with multisets?

$$\{\langle A \rangle \rightarrow 1, \langle B \rangle \rightarrow 3\} \cup \{\langle B \rangle \rightarrow 2, \langle C \rangle \rightarrow 4\} =$$

$$\{\langle A \rangle \rightarrow 1 + 0, \langle B \rangle \rightarrow ???, \langle C \rangle \rightarrow ???\}$$

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$$\{\langle A \rangle \rightarrow 1 + 0, \langle B \rangle \rightarrow 3 + 2, \langle C \rangle \rightarrow 0 + 4\}$$

—

$$\{\langle A \rangle \rightarrow 1\} \cup \{\langle A \rangle \rightarrow -1\} =$$

How does union work with multisets?

$$\{\langle A \rangle \rightarrow 1, \langle B \rangle \rightarrow 3\} \cup \{\langle B \rangle \rightarrow 2, \langle C \rangle \rightarrow 4\} =$$

$$\{\langle A \rangle \rightarrow 1 + 0, \langle B \rangle \rightarrow 3 + 2, \langle C \rangle \rightarrow 0 + 4\}$$

—

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How does union work with multisets?

$$\{\langle A \rangle \rightarrow 1, \langle B \rangle \rightarrow 3\} \cup \{\langle B \rangle \rightarrow 2, \langle C \rangle \rightarrow 4\} =$$

$$\{\langle A \rangle \rightarrow 1 + 0, \langle B \rangle \rightarrow 3 + 2, \langle C \rangle \rightarrow 0 + 4\}$$

—

$$\{\langle A \rangle \rightarrow 1\} \cup \{\langle A \rangle \rightarrow -1\} = \{\langle A \rangle \rightarrow 0\} = \{\}$$

How does cross-product work with multisets?

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$$\{\langle A, C \rangle \rightarrow ???, \langle B, C \rangle \rightarrow ???\}$$

How does cross-product work with multisets?

$$\{\langle A \rangle \rightarrow 1, \langle B \rangle \rightarrow 3\} \cup \{\langle C \rangle \rightarrow 4\} =$$

$$\{\langle A, C \rangle \rightarrow 1 \times 4, \langle B, C \rangle \rightarrow ???\}$$

How does cross-product work with multisets?

$$\{\langle A \rangle \rightarrow 1, \langle B \rangle \rightarrow 3\} \cup \{\langle C \rangle \rightarrow 4\} =$$

$$\{\langle A, C \rangle \rightarrow 1 \times 4, \langle B, C \rangle \rightarrow 3 \times 4\}$$

How does projection work with multisets?

$$\pi_{\#1} \{ \langle A, X \rangle \rightarrow 1, \langle A, Y \rangle \rightarrow 2, \langle B, Z \rangle \rightarrow 5 \} =$$

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How does projection work with multisets?

$$\begin{aligned} \pi_{\#1} \{ \langle A, X \rangle \rightarrow 1, \langle A, Y \rangle \rightarrow 2, \langle B, Z \rangle \rightarrow 5 \} = \\ \{ \langle A \rangle \rightarrow 1, \langle A \rangle \rightarrow 2, \langle B \rangle \rightarrow 5 \} = \{ \langle A \rangle \rightarrow 3, \langle B \rangle \rightarrow 5 \} \end{aligned}$$

How does selection work with multisets?

$$\sigma_A\{\langle A, X \rangle \rightarrow 1, \langle A, Y \rangle \rightarrow 2, \langle B, Z \rangle \rightarrow 5\} =$$

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$$\sigma_A\{\langle A, X \rangle \rightarrow 1, \langle A, Y \rangle \rightarrow 2, \langle B, Z \rangle \rightarrow 5\} =$$

$$\{\langle A, X \rangle \rightarrow 1, \langle A, Y \rangle \rightarrow 2, \langle B, Z \rangle \rightarrow 0\} \{\langle A, X \rangle \rightarrow 1, \langle A, Y \rangle \rightarrow 2\}$$

$$\mathbb{K} = \mathbb{X}$$

$$\pi, U = +$$