Approximate Query Processing

CSE 4/562: Database Systems | Lecture 13

Query rewrites are **correct** when they replace one part of the plan with another **guaranteed to compute the same result**

A little bit of error is ok sometimes...

Error-Tolerant Use Cases

- Cardinality estimation for join order selection
- Cloud resource provisioning
- Business intelligence dashboards
- Spam Filtering
- Network Monitoring
- Machine Learning (Bayes Nets, Neural Nets)

Sketches

```
SELECT approx_count_distinct(A), approx_top_k(B)
FROM BIG_TABLE
```

Lossy state encodings for “hard” (holistic) aggregates.

Trading accuracy for speed...


```
SELECT listagg(A)
FROM R
```

$$[3, 6, 10, 9, 1, 3, 9, 7, 9, 4, 7, 9, 2, 1, 2, 4, 10, 8, 9, 7]$$

Trading accuracy for speed...


```
SELECT listagg(A)
FROM R
```

$$[3, 6, 10, 9, 1, 3, 9, 7, 9, 4, 7, 9, 2, 1, 2, 4, 10, 8, 9, 7]$$

—

```
SELECT avg(A)
FROM R
```

$$6$$

Trading accuracy for speed...


```
SELECT listagg(A)
FROM R
```

$[3, 6, 10, 9, 1, 3, 9, 7, 9, 4, 7, 9, 2, 1, 2, 4, 10, 8, 9, 7]$

—

```
SELECT avg(A)
FROM R
```

6

—

```
SELECT avg(A), listagg(",", A)
FROM R
WHERE rand() < .25
```

$5.8, [3, 6, 10, 9, 1]$

Samples

- $\text{avg}(\text{sample}(A, \text{rate})) \approx \text{avg}(A)$
- $\frac{\text{sum}(\text{sample}(A,\ \text{rate}))}{\text{rate}} \approx \text{sum}(A)$

Sampling lets you approximate aggregate values with less data

Did I generate enough samples?

Estimation error

- "The correct answer is definitely in $[a, b]$"
  - Maximum safety
  - Not always possible without seeing all the data
- "I'm at least 95% sure the answer is in $[a, b]$"
  - Viable for sampling

"I'm at least $100 \cdot (1 - \delta)\%$ sure the answer is within ε."

Hoeffding's Bound

With S being n tuples sampled uniformly *with* replacement from any R:

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$$|\text{avg}(S) - \text{avg}(R)| \quad \text{The absolute error}$$

Hoeffding's Bound

With S being n tuples sampled uniformly *with* replacement from any R:

$\lvert\text{avg}(S) - \text{avg}(R)\rvert$	The absolute error
$P(\lvert\text{avg}(S) - \text{avg}(R)\rvert \geq \varepsilon)$	The probability of exceeding error threshold ε

Hoeffding's Bound

With S being n tuples sampled uniformly *with* replacement from any R:

$$|\text{avg}(S) - \text{avg}(R)|$$

The absolute error

$$P(|\text{avg}(S) - \text{avg}(R)| \geq \varepsilon)$$

The probability of exceeding error threshold ε

$$P(|\text{avg}(S) - \text{avg}(R)| \geq \varepsilon) \leq 2e^{\frac{2n\varepsilon^2}{(\max(R) - \min(R))^2}}$$

... is at worst some threshold

Hoeffding's Bound

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"Hoeffding's Bound" (Generalizable to sum, count, stddev, etc...)

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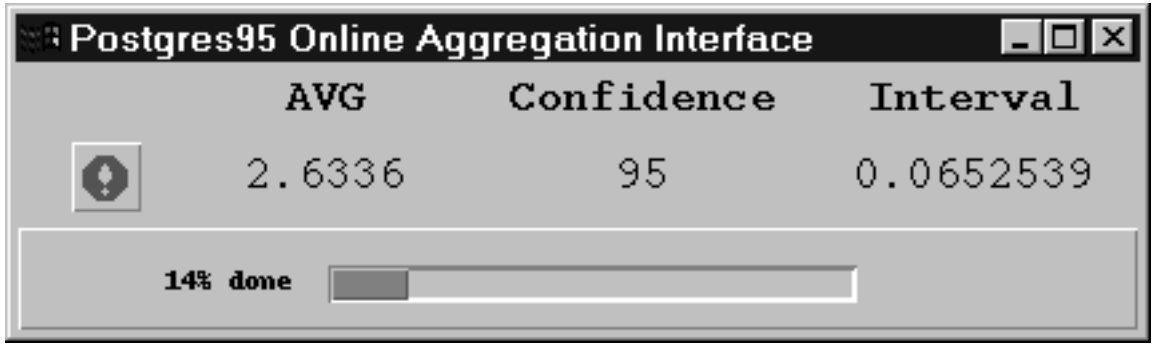
"Hoeffding's Bound" (Generalizable to sum, count, stddev, etc...)

See also "Chernoff's Bound", "Serfling's Bound" and others.

How do we use it?



Interfaces

- Keep adding samples until you reach a target accuracy
- Keep adding samples until you run out of time
- Keep adding samples until the user tells you to stop

Image credit: 'Online Aggregation', Hellerstein et. al.

Generating samples.

What is hard about generating samples?

What is hard?

Sampling From Disk

- Random seeks are slow

Sampling for Group By or Filtering

- Low frequency events don’t get sampled

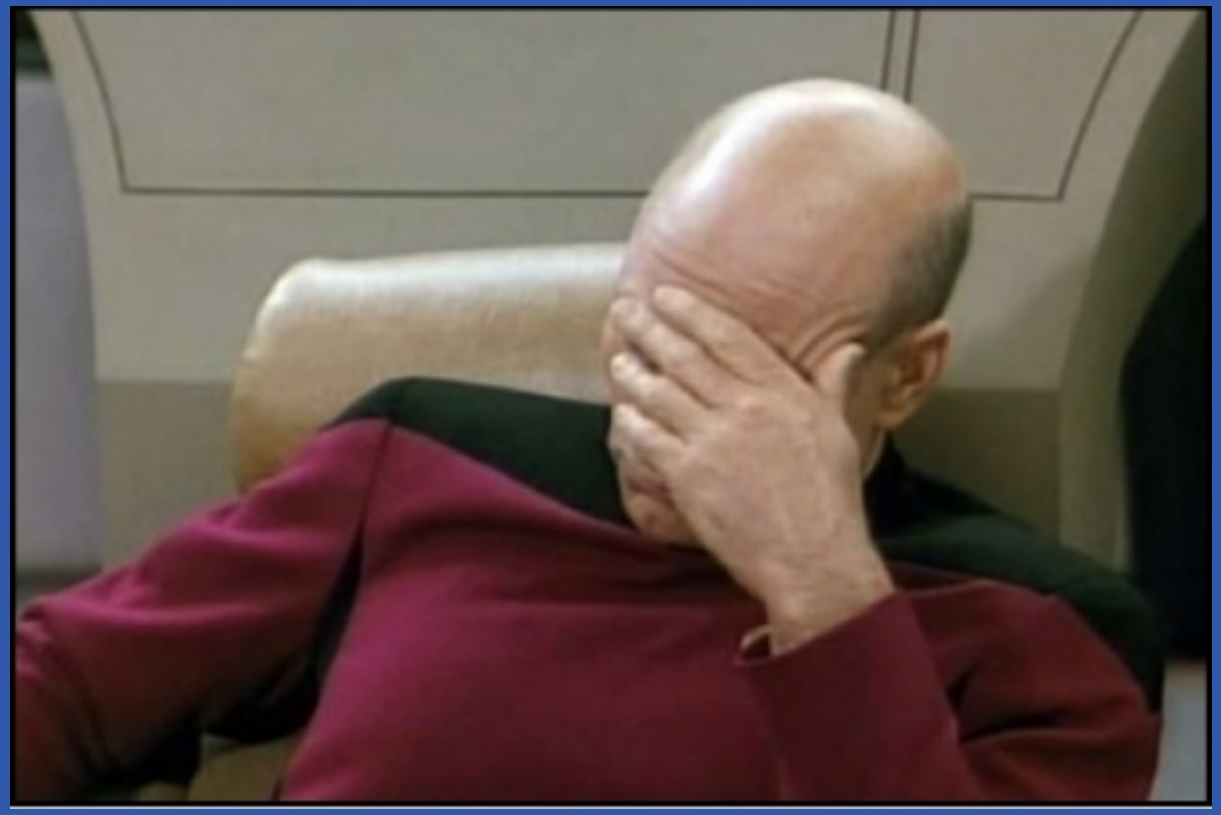
Sampling from Joins

- The Birthday Paradox

Sampling from disk

Idea 1: Pick randomly!

```
for i in range(num_samples):
      sample_id = random(0, num_records)
      samples += [ table.where( rowid = sample_id ) ]
```



Problem: Random reads have low spatial locality.

Idea 2: Sequential Scan.

```
sample_ids = [
  random(0, num_records)
  for i in range(num_samples)
]
samples = table.join(sample_ids)
```

Problem: This requires knowing how many records there are.

Idea 3: Reservoir Sampling.

```
i = 0
reservoir = []
for row in table:
  if len(reservoir) < TARGET:
    reservoir += row
  else if rand() < TARGET / len(reservoir):
    reservoir[rand_int() % TARGET] = row
```

Sampling from Group-By

Group-By Sampling

Employee	City	Salary
Alice	NYC	$120k
Bob	NYC	$110k
Carol	NYC	$115k
Dave	Syracuse	$80k

```
SELECT City, AVG(Salary) FROM NYS_Salaries;
```

Group-By Sampling

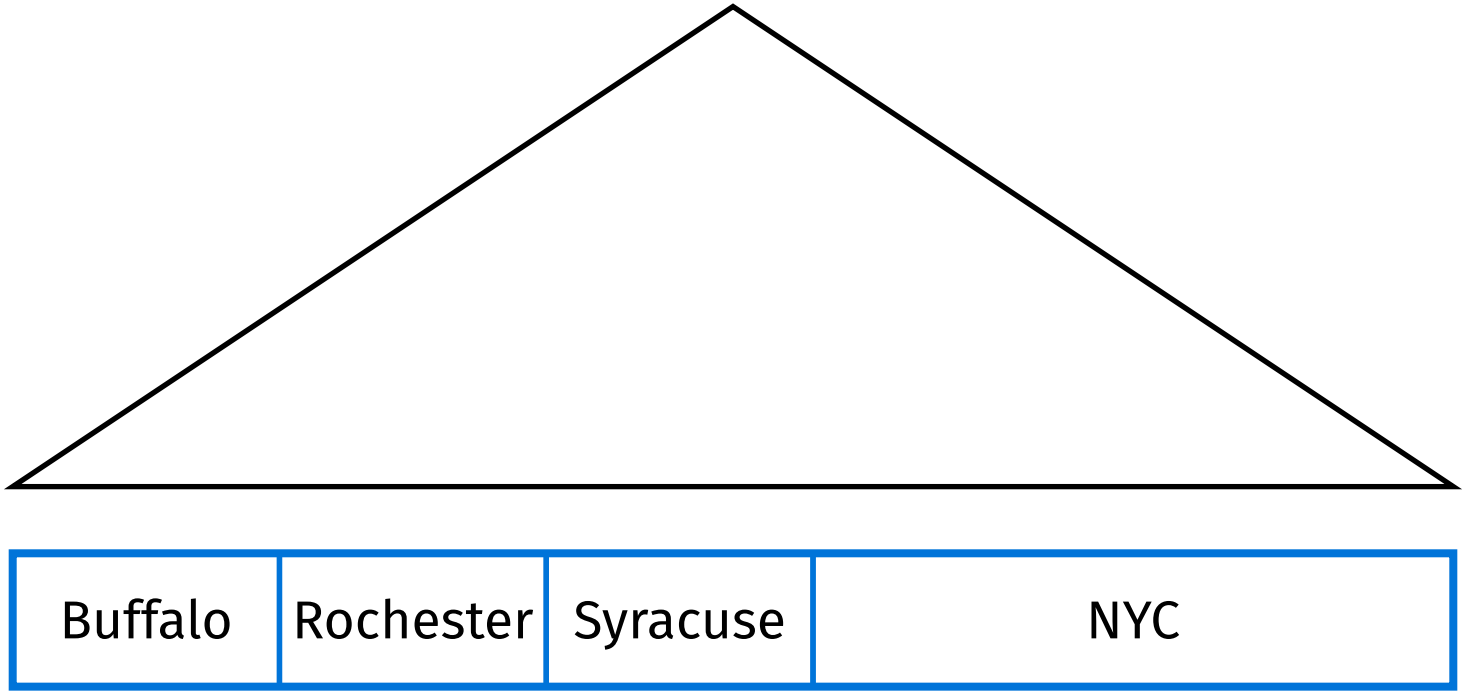
Problem: Most data is about NYC. With N samples taken uniformly, margins of error for other cities are much bigger.

Stratified Sampling

Generate $\frac{N}{\texttt{COUNT(DISTINCT City)}}$ samples for each group

Use $\texttt{COUNT(DISTINCT City)}$ instead of $\texttt{COUNT}(*)$ as the total group size

Index Striding



'Online Aggregation', Hellerstein et. al.

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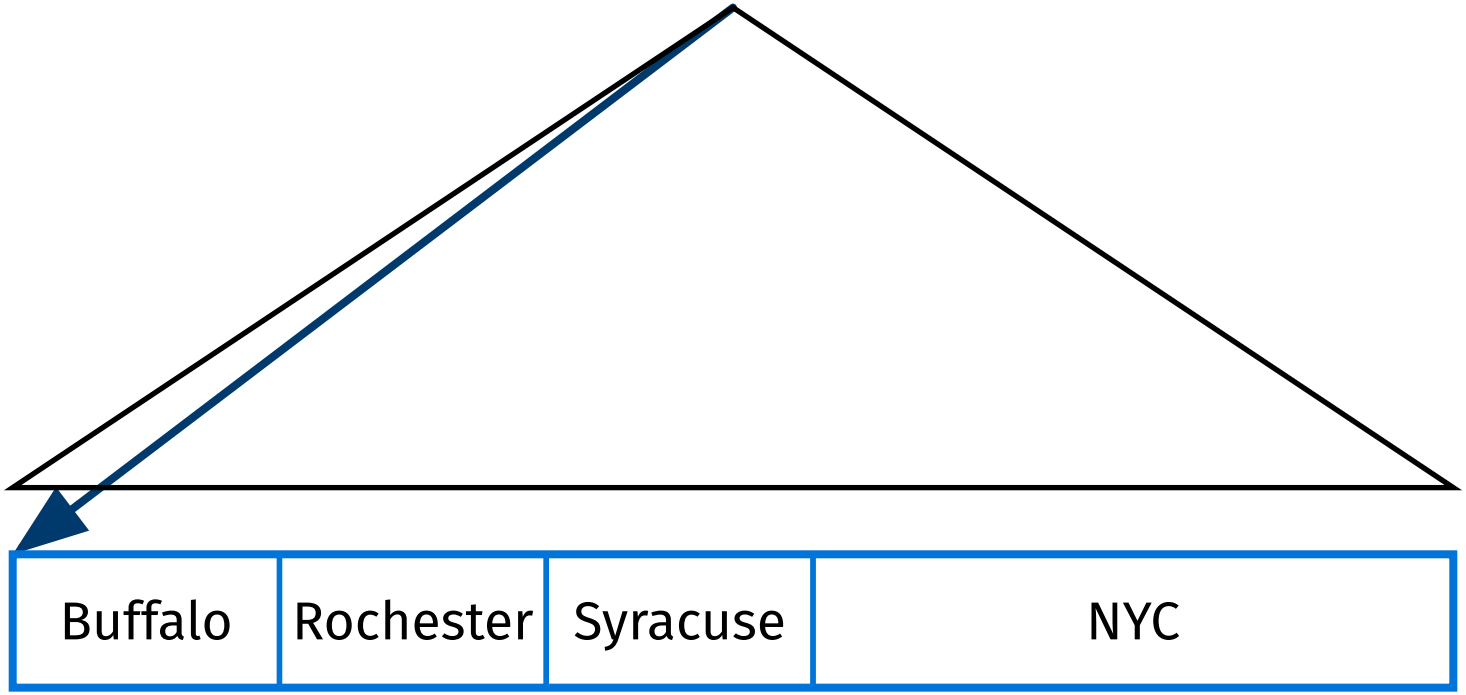

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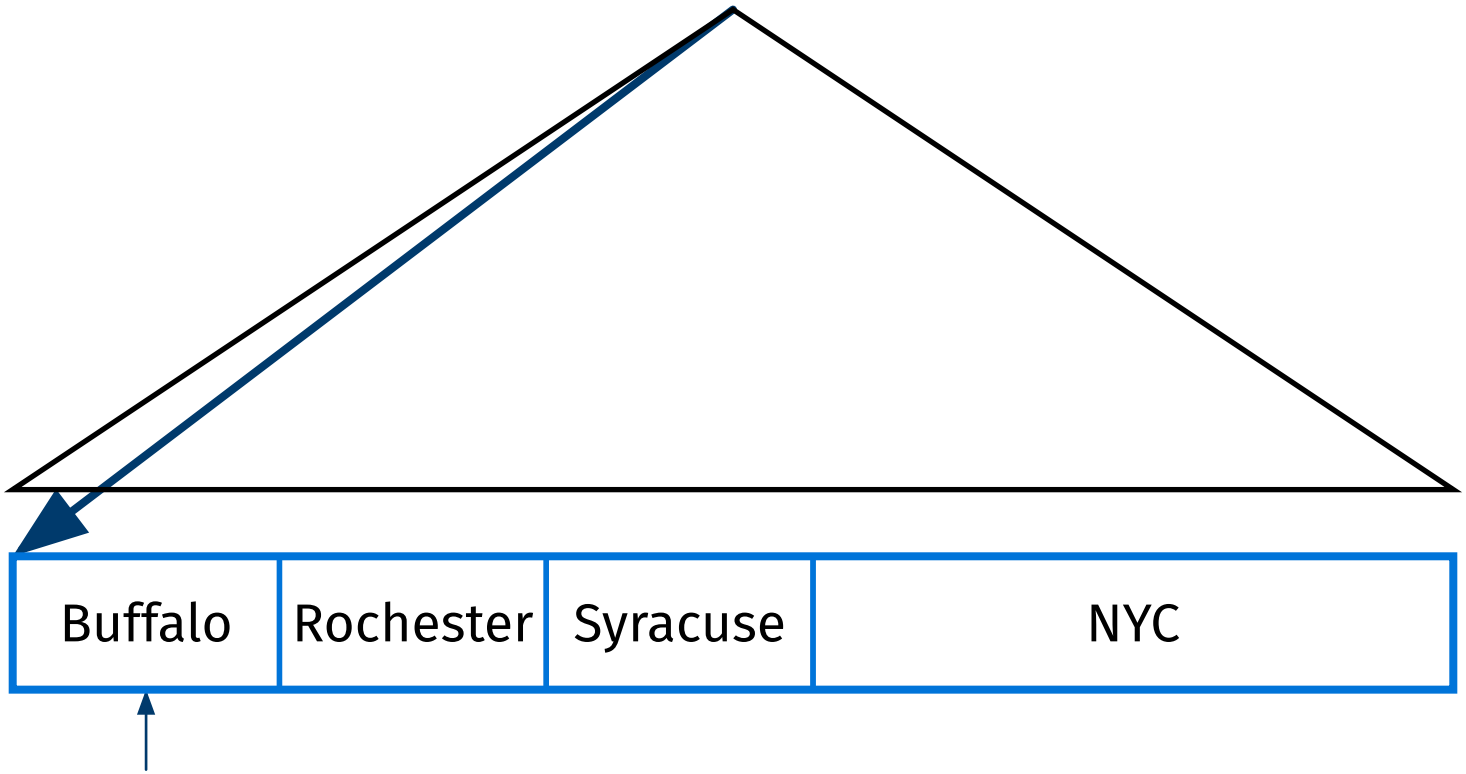

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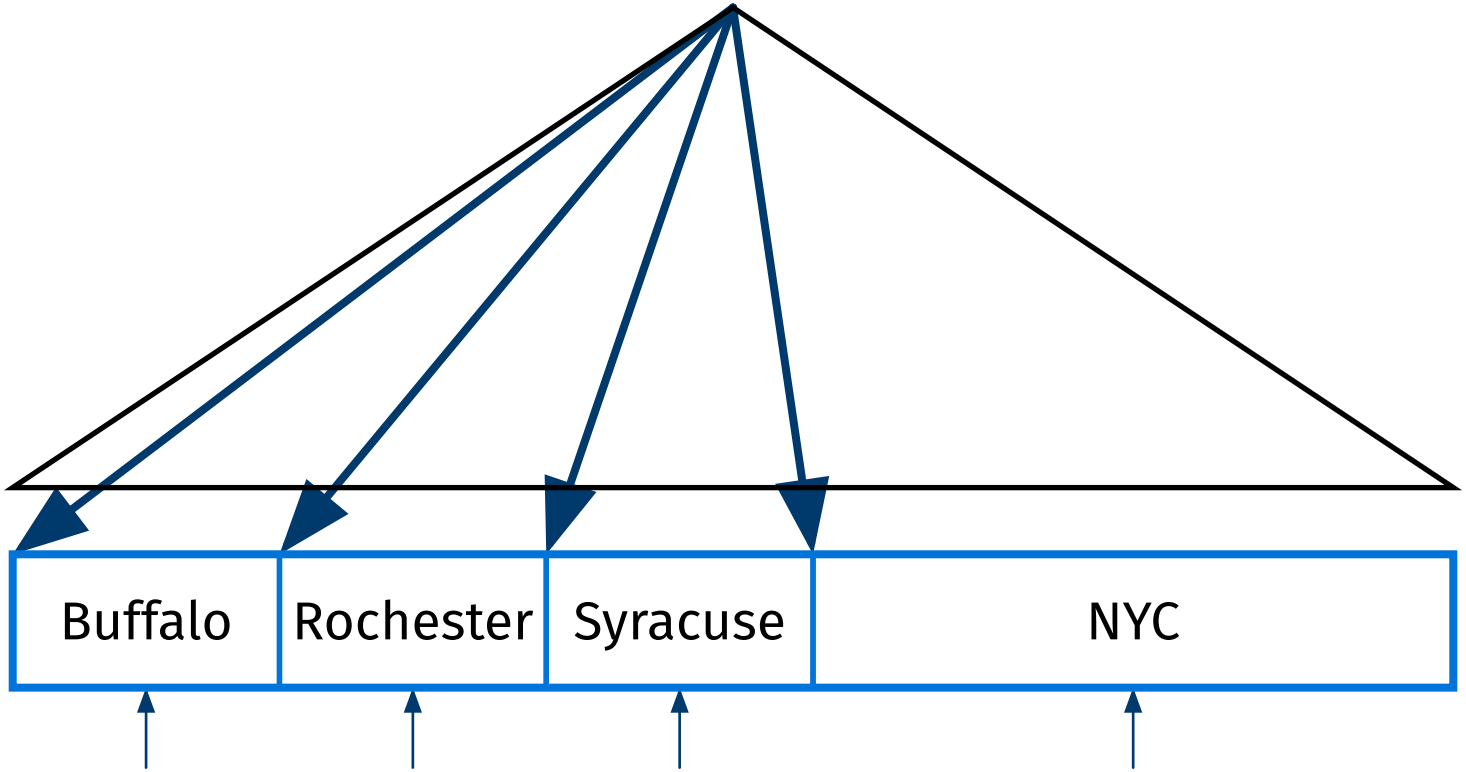

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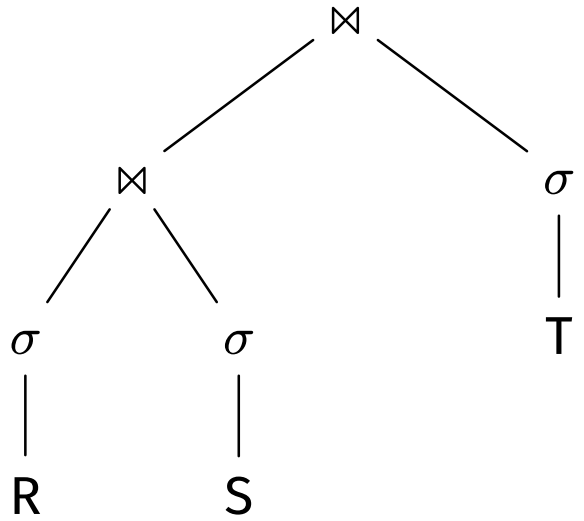
Sampling from Joins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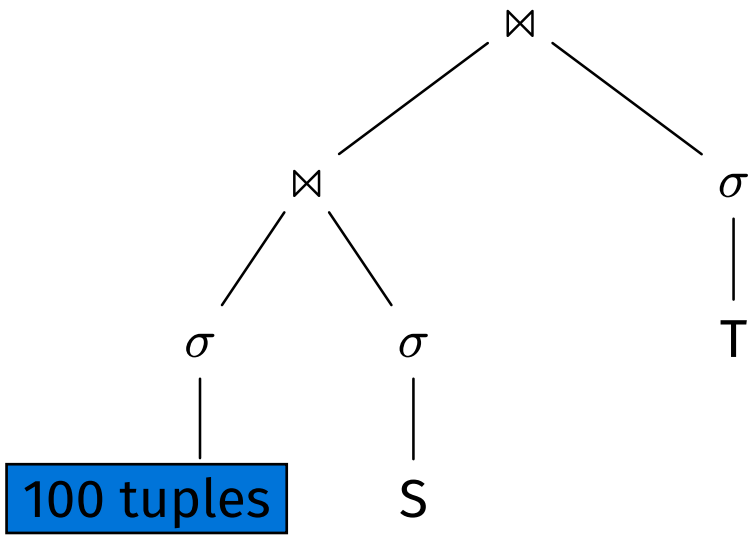
The Birthday Paradox



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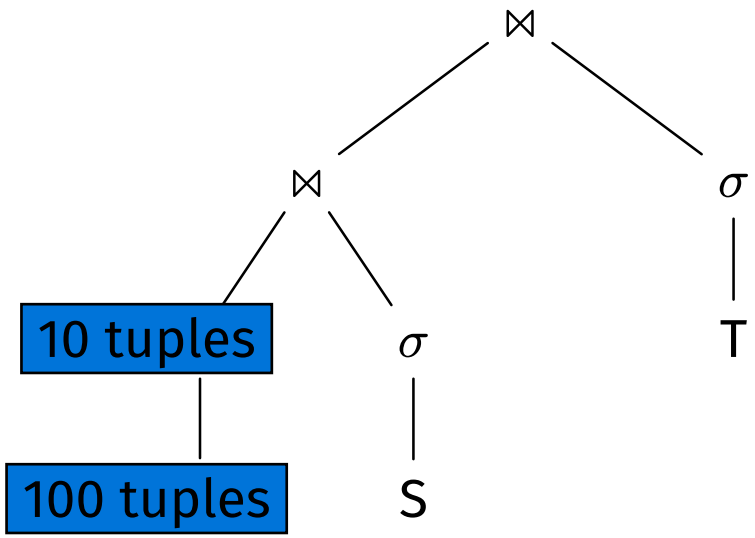

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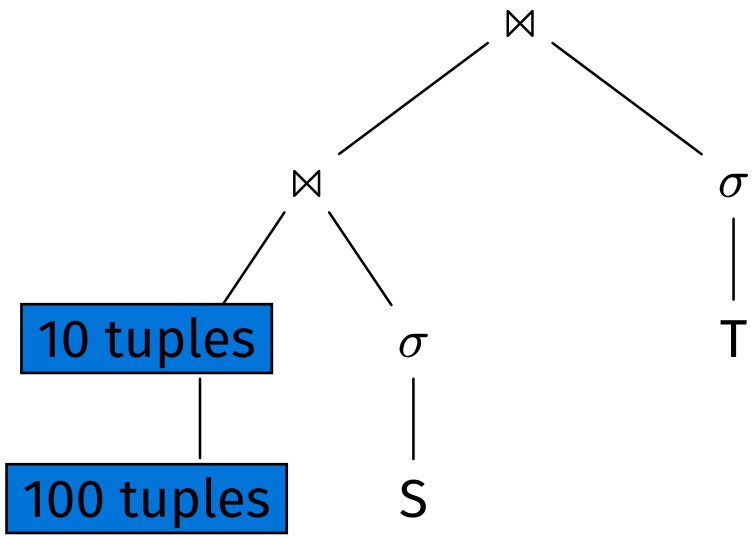

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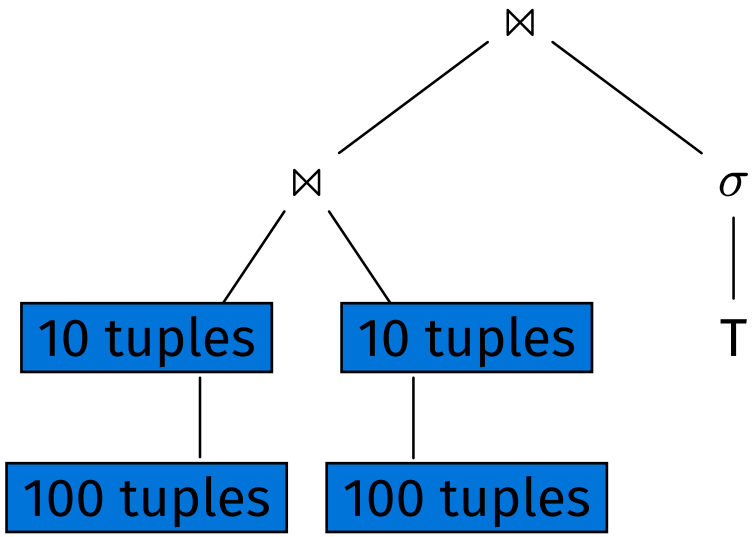

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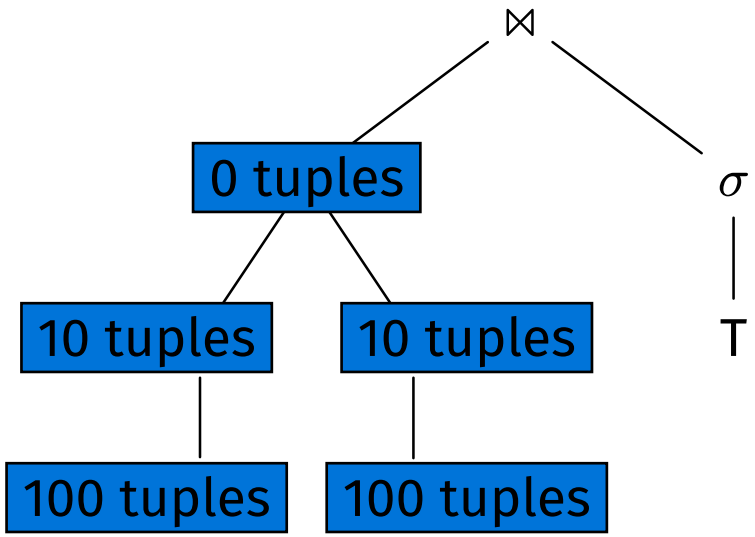
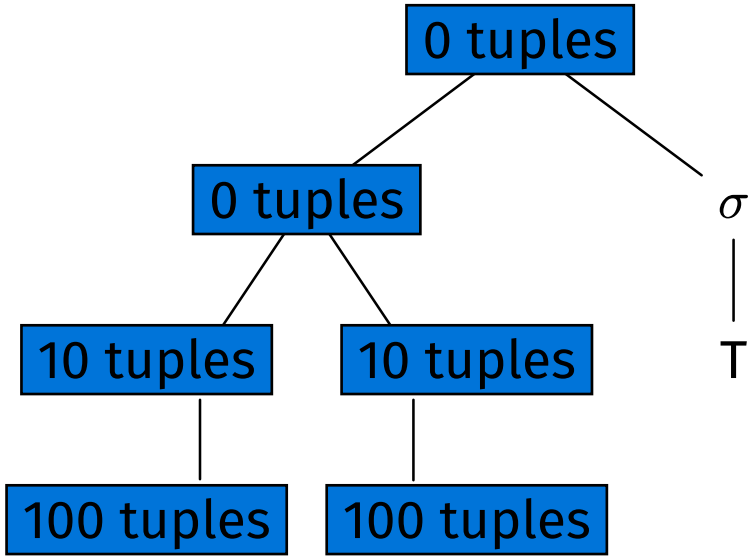



0 tuples
0 tuples
σ
10 tuples
10 tuples
T
100 tuples
100 tuples

Assume: `UNIQ(A,R) = UNIQ(A,S) =` N

The Birthday Paradox

It takes $O\left(\sqrt{N}\right)$ samples from `R` and `S` each to expect to get even one match.

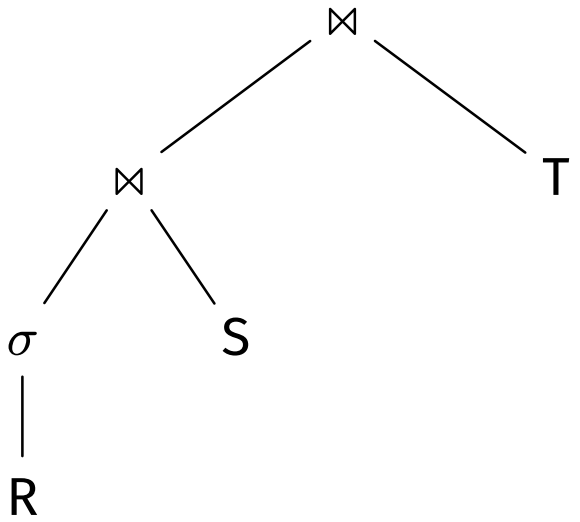
Join Synopses

Given $R \underset{B}{\bowtie} S \underset{C}{\bowtie} T$ where

- $R.B$ is a foreign key reference to $S.B$
- $S.C$ is a foreign key reference to $T.C$

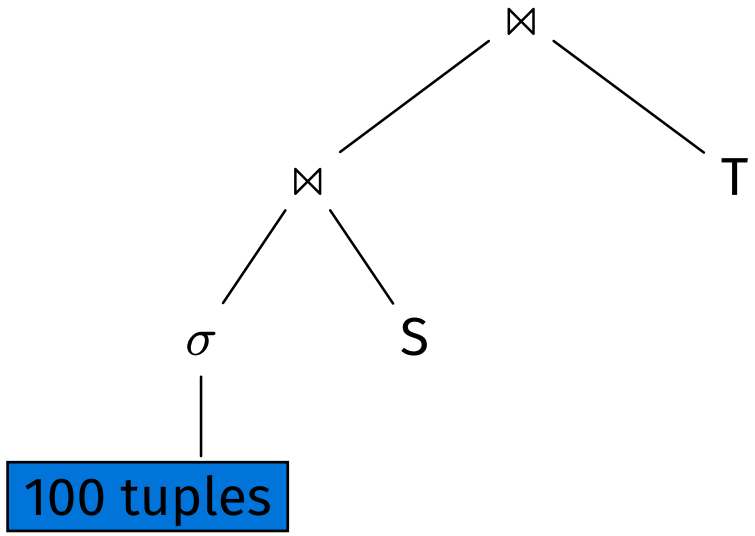
Idea: Sample from R, use all of S, T

Join Synopses



'Join synopses for approximate query answering', Acharya et. al.

Join Synopses



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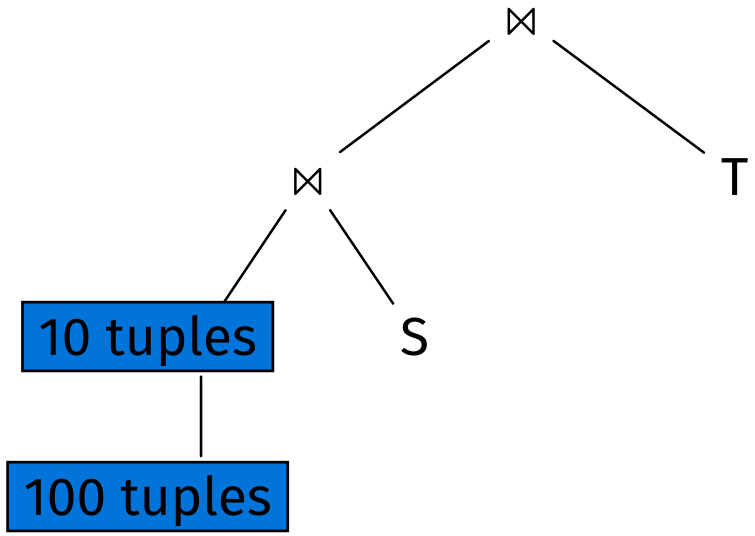

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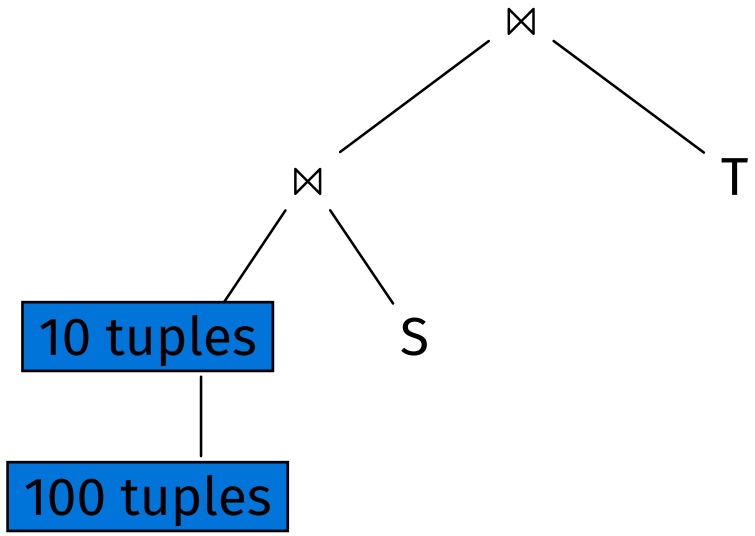

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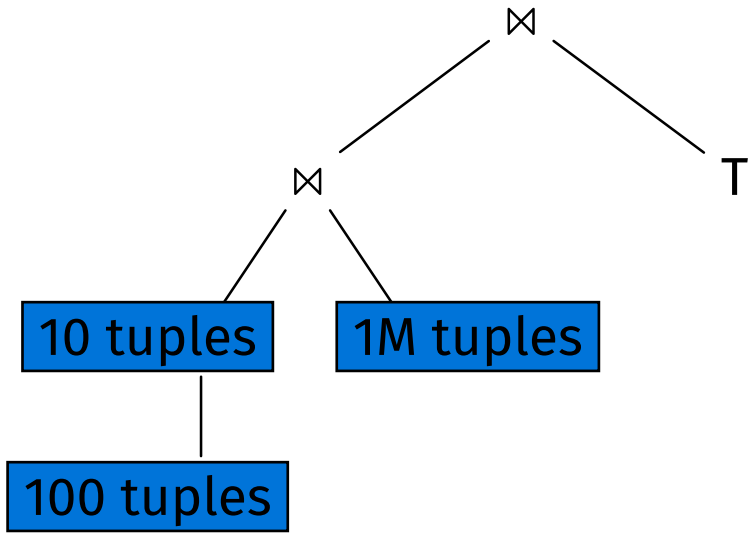

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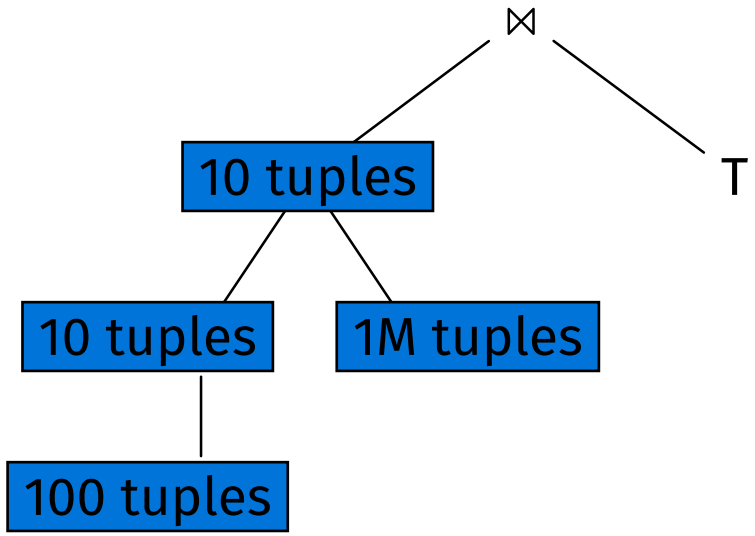

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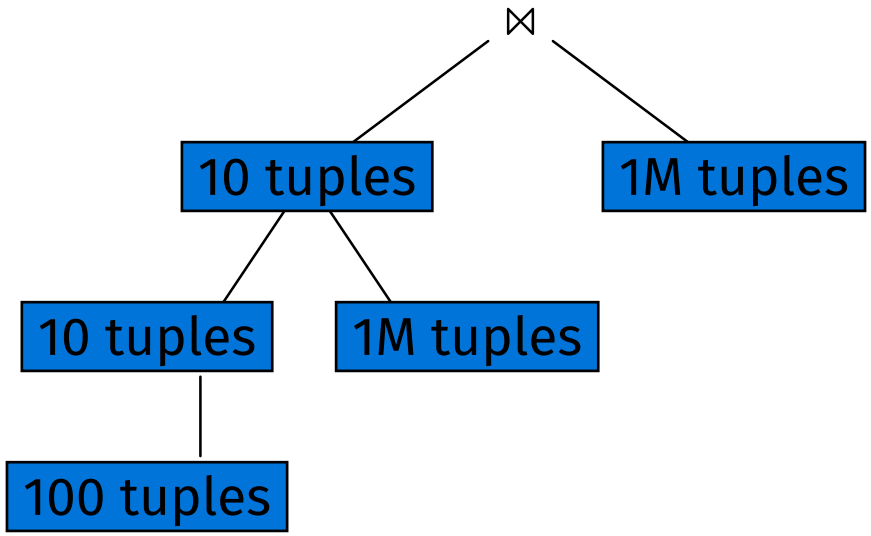

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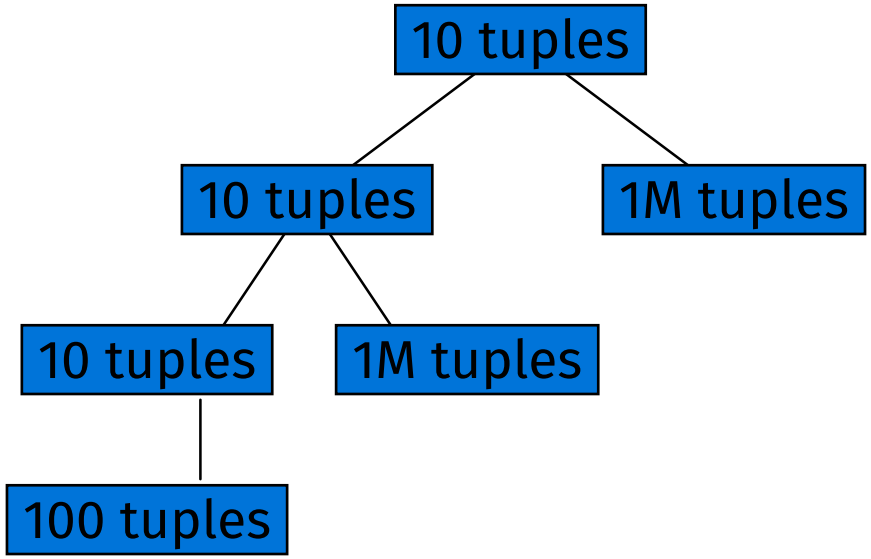

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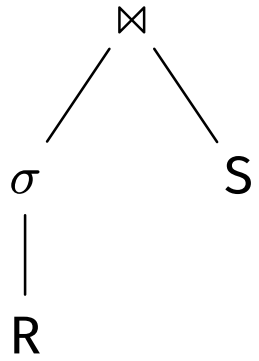
Idea 2: Use stratified sampling



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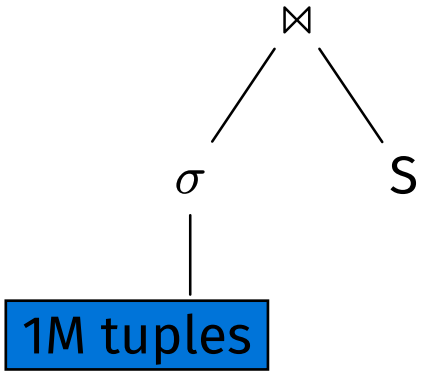

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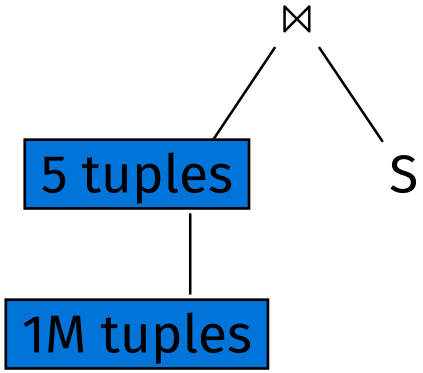

Stratified Sampling

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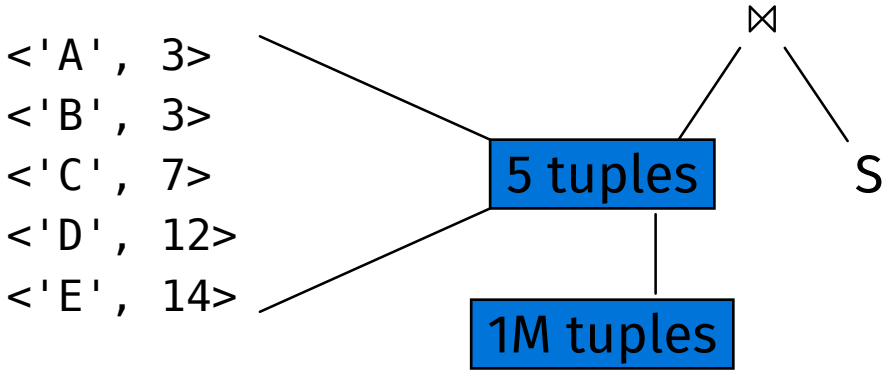

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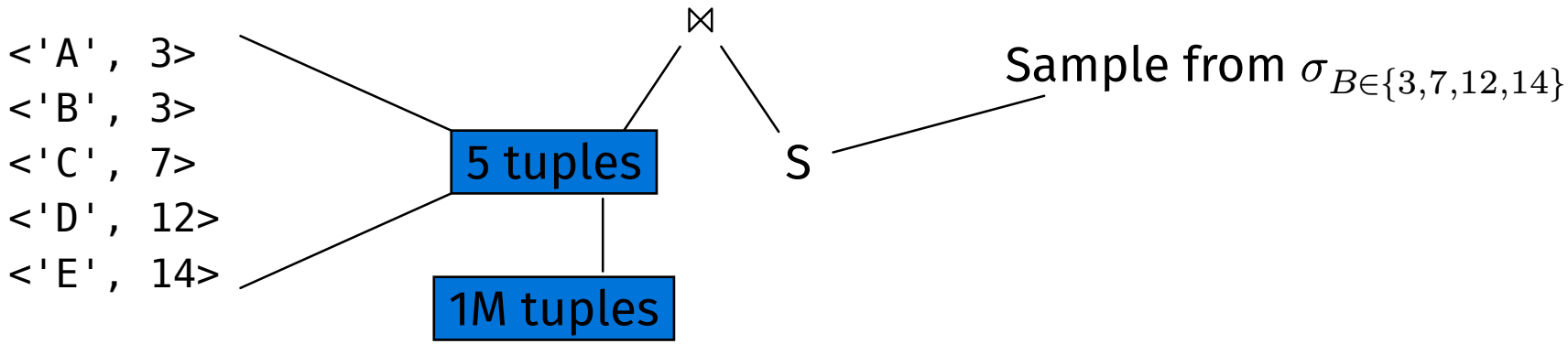

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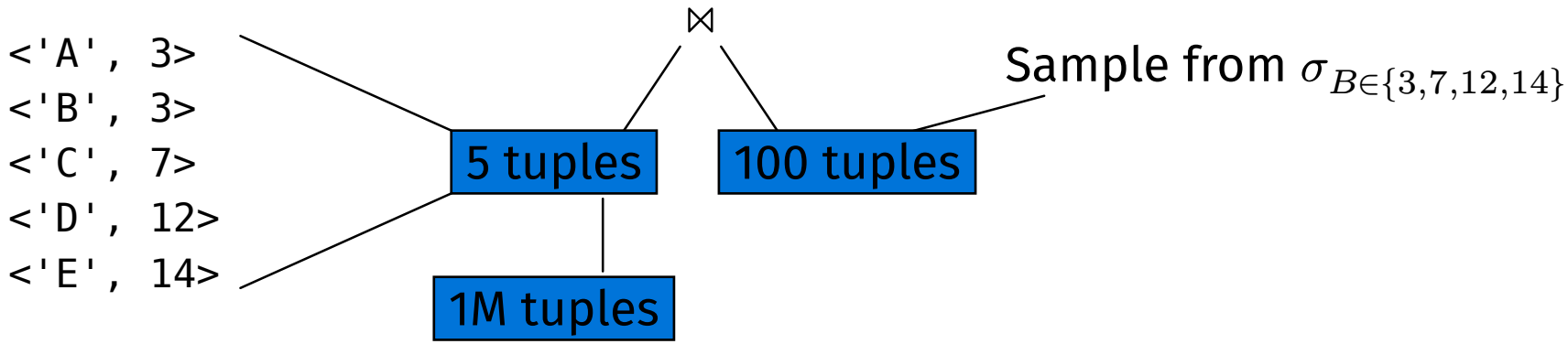

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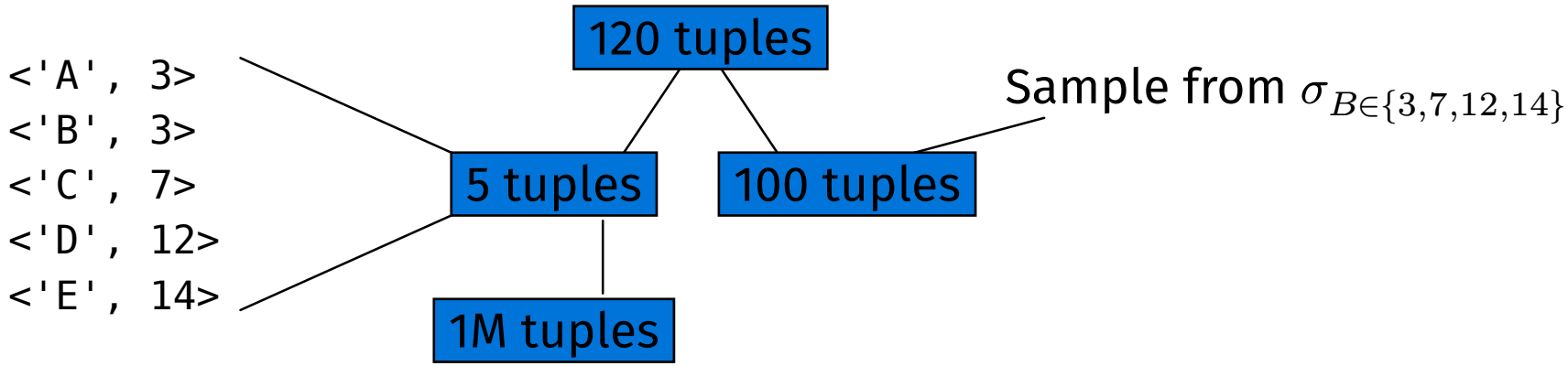


Are we biasing the sampling process?

Goal: For some $r \in R$ and $s \in S$...

Sample $r \bowtie s$ with probability $P((r \bowtie s) \in R \bowtie S)$

—

Actual: Sample a tuple from R, then sample a tuple from S based on R

$P(r \in R) \cdot P((r \bowtie s) \in R \bowtie S \mid r \in R)$

Bayes Theorem

$$P(r \in R) \cdot P((r \bowtie s) \in R \bowtie S \mid r \in R)$$

$$= P(r \in R \wedge (r \bowtie s) \in R \bowtie S)$$

$$\neq P((r \bowtie s) \in R \bowtie S)$$

Corrective Factor

Find a value C such that...

$$P(r \in R \wedge (r \bowtie s) \in R \bowtie S) \cdot C = P((r \bowtie s) \in R \bowtie S)$$

$$C = \frac{P((r \bowtie s) \in R \bowtie S)}{P(r \in R \wedge (r \bowtie s) \in R \bowtie S)}$$

$$= \frac{1}{P(r \in R \mid (r \bowtie s) \in R \bowtie S)} \approx |R| \cdot |\{s \mid r \text{ joins with } s \in S\}|$$

(i.e., the probability that R participates in **some** join)

Sampling Order

Should I sample from R or S first?

- What if R is much bigger than S?
- What if only one tuple of R joins with a tuple from S?
- What if some filtering predicate removes the tuple from S that R joins with?

'Wander Join: Online Aggregation via Random Walks'; Li et. al.

(Ask Zhuoyue Zhao if you're interested)

Ripple Join

- Incrementally increase the sample size

Leaky Joins

- Rotate through the space of possible join results

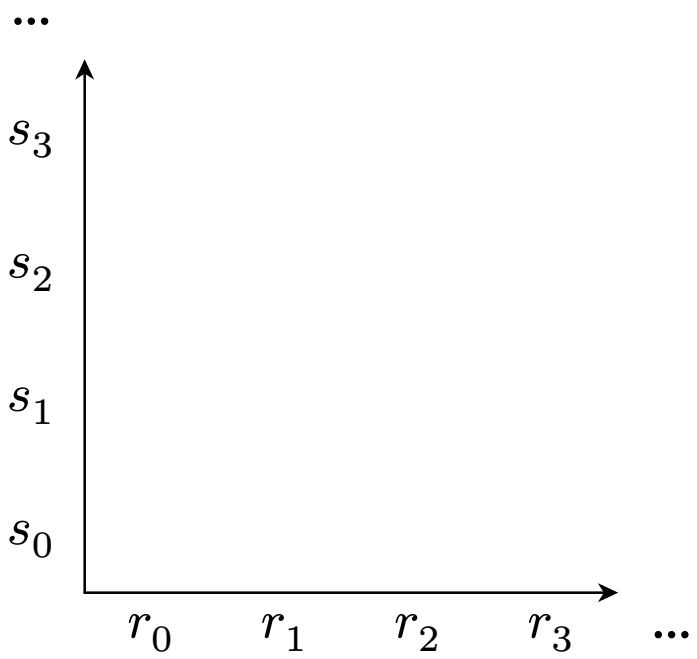
Ripple Join

$$Q = R \bowtie S$$

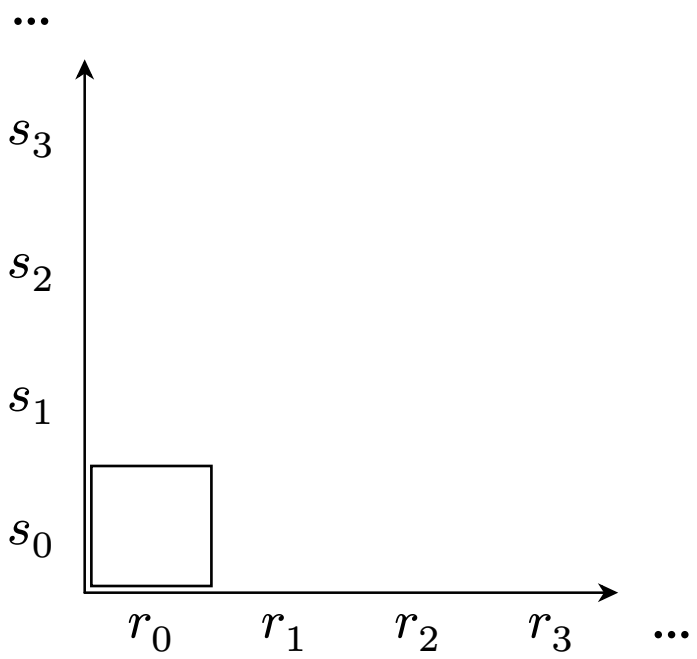
```
Q = []; r_sample = []; s_sample = []

while not r.done() and not s.done()
  if r_current := r.next_sample() is not None:
    Q += [ r_current ] ⋈ s_sample
    r_sample += [ r_current ]

  if s_current := s.next_sample() is not None:
    Q += [ s_current ] ⋈ r_sample
    s_sample += [ s_current ]
```


...
s_3
s_2
s_1
s_0
r_0
r_1
r_2
r_3
...

Ripple Join



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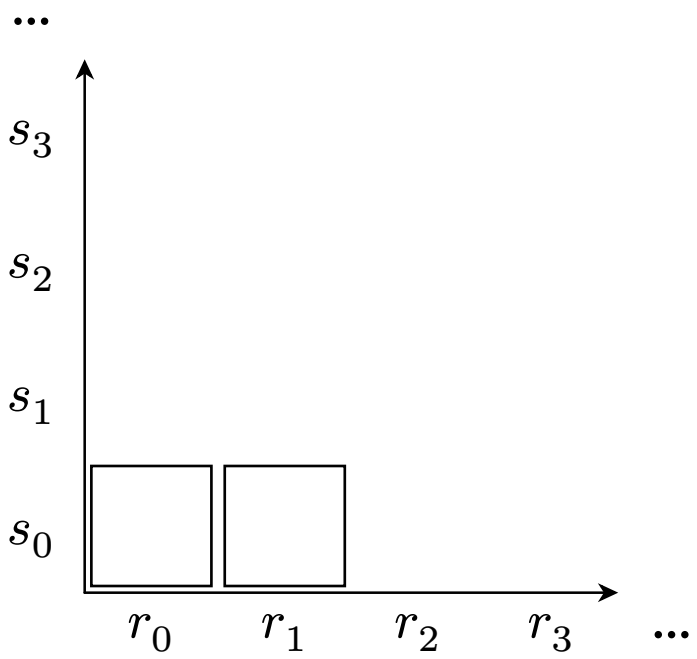

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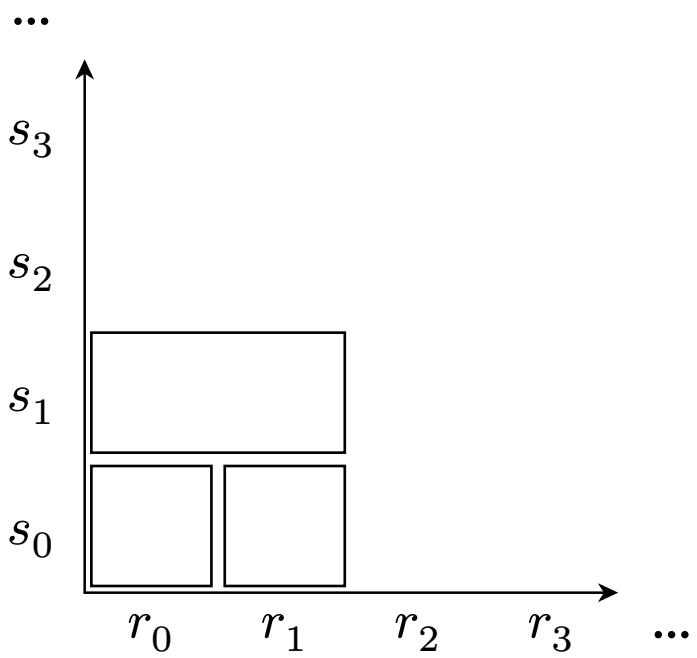

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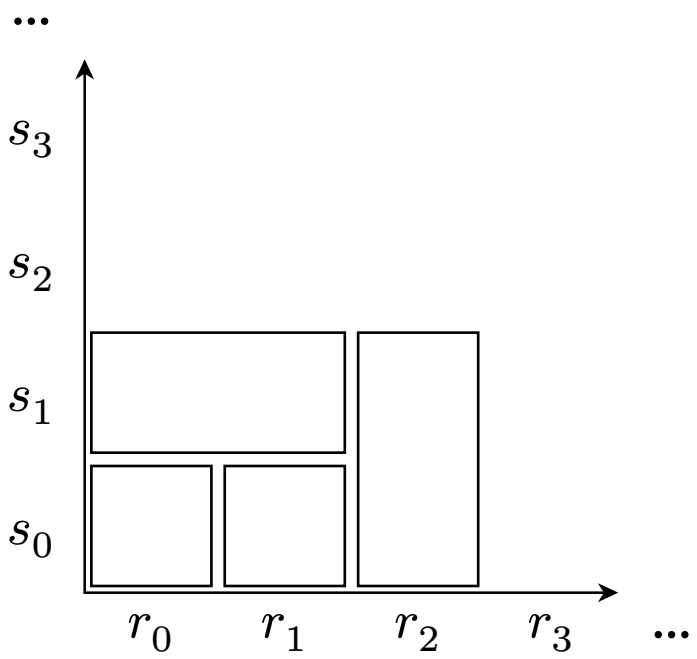

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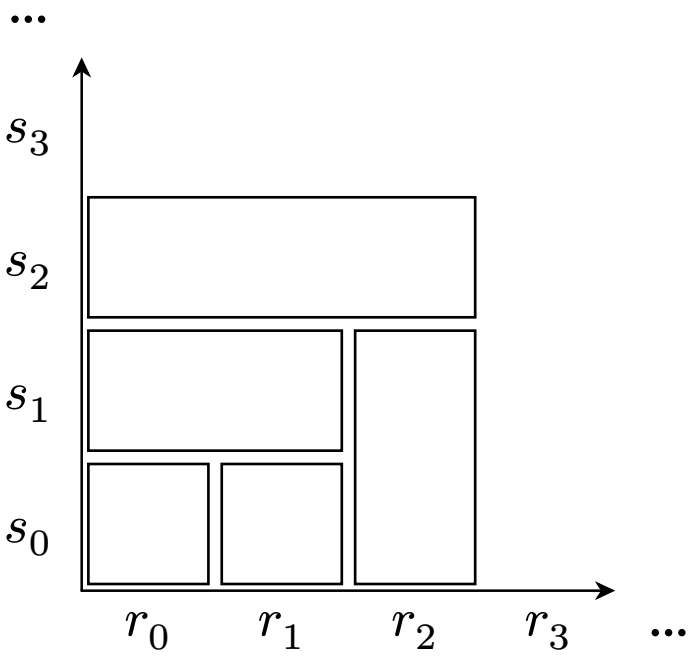

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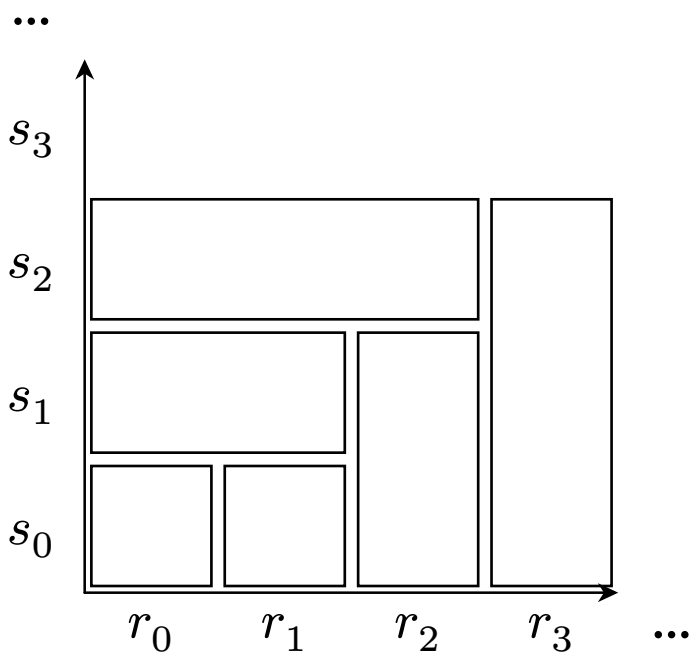

Output-first Sampling

For $Q = \pi_{R.A,R.B,S.C}\left(R \underset{B}{\bowtie} S\right)$

1. Pick a random tuple <a, b, c>
2. Test for <a,b> in R
3. Test for <b,c> in S
4. If both are present, add <a, b, c> to the sample.
5. Repeat as needed

Dense joins.

- Inference in Bayes Nets
- (Dense) Linear Algebra

Output-first Sampling

Problem:

- Sampling without replacement will never converge
- Sampling with replacement means keeping N^k state for k attributes

Linear Congruential Generators

$X_n = (aX_{n-1} + b) \bmod m$ where...

- m and b are relatively prime
- $a - 1$ is divisible by all prime factors of m
- $a - 1$ is divisible by 4 if m is divisible by 4

Then for any $0 \leq X_0 < m$...

$\{X_0, ..., X_{m-1}\}$ is a pseudorandom permutation of [0, m)

The Leaky Join Operator


```
# m = |dom(A1)| * |dom(A2)| * ... * |dom(Ak)|
(a, b) = init_lcg(m)
x = rand_int() % m

for i in range(m):
  x = (a * x + b) % m
  t_left, t_right = extract(x)

  if left.contains(t_left):
    if right.contains(t_right):
      yield t_left + t_right;
```